6 Rules of Factoring:
1. **GCF** (any # terms)
   - Greatest Common Factor
2. **Difference of 2 Perfect Squares** (2 terms)
   - Memorize pattern
3. **Sums or Differences of Cubes** (2 terms)
   - Memorize pattern
4. **Trinomials** (3 terms)
   - Box, Fraction, Trial & Error
5. **Grouping** (4 terms)
   - GCF times 3
6. **Factor Completely** (any # Terms)
   - Factor until you can factor no more

\[(x+3)(x+2)\]
Rule # 1: **GCF** (any # terms)

- Find something that all terms share or have in common.
- Divide each term by the GCF.
- Remember to Subtract Exponents!
- **GCF MUST** be written in front for all expressions

\[ 4x^3+8x^2-6x^5-12x: \quad 2x (2x^2+4x-3x^4-6) \]

\[ 4(x+5) - x(x+5): \quad (x+5)(4-x) \]
Rule # 2: **Difference of 2 Perfect Squares** (2 terms)

- A very simple pattern - Memorize
- 1\textsuperscript{st} term is a perfect square, Last term is a perfect square
  There is NO MIDDLE TERM!
- Square root of each term - one positive & one negative

\[(x^2 - y^2): \quad (x + y)(x - y)\]

\[
\begin{align*}
a^2 &- 49 & (a - 7)(a + 7) \\
b^2 &- 144 & (b + 12)(b - 12) \\
16x^2 &- 324 & (4x - 18)(4x + 18) \\
4(4x^2 &- 81) & 4(2x - 9)(2x + 9) \\
a^2 &+ 36 & \text{GOTCHA!}
\end{align*}
\]
Rule # 3: Sums or Differences of Cubes (2 terms)

- A very simple pattern - Memorize
  1) Find the base root of both terms - cube root
     Then follow the magic pattern:
  2) Square the 1st term - of the base root
  3) Multiply the base roots together and change the sign +/-
  4) Square the last term - of the base root

\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]
\[ a^3 - b^3 = (a - b)((a^2 + ab + b^2)) \]
\[ 27x^3 + 64y^3 = (3x + 4y)((9x^2 - 12xy + 16y^2)) \]
Rule # 2: **Trinomials** (3 terms)

- The following methods will all work with trinomials:
  - The BOX, Fraction Method, Trial & Error/(Guess & Check) or Grouping

**Sum/Part/Product & Trial & Error**
Old standby's: they work great if you know how to factor very well.

**Alternative Methods - Box, Fraction, & Grouping**
For these methods you must make a Product|Sum chart.

To make a Product|Sum chart:

A) Find the product & the sum: \( x^2 - 8x + 15 \)
   Multiply the outside terms for the product \( (15x^2) \)
   The middle term is the sum \(-8x\)

B) To find the parts - Play the Product & Sum Game:
   Ask yourself \_\_?\_ times \_\_?\_ = the product,
   but adds/subtracts to be the sum?

\[ x^2 - 8x + 15 \]

The product is \(15x^2\), the sum is \(-8x\)

List all the factors of the product until you come up with the pair that make the correct sum

<table>
<thead>
<tr>
<th>Product</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(15x^2)</td>
<td>(-8x)</td>
</tr>
<tr>
<td>(15x, x)</td>
<td>(14x) N</td>
</tr>
<tr>
<td>(-15x, -x)</td>
<td>(-16x) N</td>
</tr>
<tr>
<td>(5x, 3x)</td>
<td>(8x) N</td>
</tr>
<tr>
<td>(-5x, -3x)</td>
<td>(-8x) Y</td>
</tr>
</tbody>
</table>
Once you have found the parts, you may plug them into one of the three methods below.

The BOX, Fraction, or Grouping Method.

The **BOX Method**: A great organizational tool.

A) 1st term - first box  
(top left corner)  \(x^2\)

B) Last term - last box  
(bottom right corner)  15

C) New parts go in the diagonals in any order.  \(-3x, 5x\)

\[
\begin{array}{c|c}
\hline
x^2 & -5x \\
\hline
-3x & 15 \\
\hline
\end{array}
\]

Now find the GCF of each row (L to R) & column (T to B)  
Use the sign of the first number you see in order.

\[x^2 - 8x + 15\]

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<td>-8x Y</td>
</tr>
</tbody>
</table>

\(G\)

\[
\begin{array}{c|c}
\hline
x^2 & -5x \\
\hline
-3x & 15 \\
\hline
\end{array}
\]

\(x\)  
\((-3)(x-5)\)
Factor: $6x^2 + 13x + 6$

$$\begin{array}{c|c}
6x^2 & 9x \\
4x & 6 \\
\end{array}$$

Prod | Sum
---|---
$36x^2$ | $13x$
$9x \& 4x$

$(3x + 2)(2x + 3)$

Factor: $12x^2 - 7x - 10$

$$\begin{array}{c|c}
12x^2 & -15x \\
4x & 10 \\
\end{array}$$

Prod | Sum
---|---
$120x^2$ | $-7x$
$-15x \& 8x$

$(3x + 2)(4x - 5)$
The **Fraction Method**: A newer quicker method.

- Make a Product / Sum Chart
- Write down the 2 new parts
- Underneath the parts, write the 1st term
- Reduce the fraction
- Write the answer from the bottom up

\[
\begin{array}{c|c}
\text{Prod} & \text{Sum} \\
\hline
15x^2 & -8x \\
\end{array}
\]

\[
\frac{-3x}{x^2} & \frac{-5x}{x^2} \\
& \frac{-3}{x} & \frac{-5}{x} \\
\end{array}
\]
\[
(x - 3)(x - 5)
\]

Factor: \(6x^2 + 13x + 6\)

\[
\begin{array}{c|c}
\text{Prod} & \text{Sum} \\
\hline
36x^2 & 13x \\
\end{array}
\]

\[
\frac{9x}{6x^2} & \frac{4x}{6x^2} \\
& \frac{3}{2x} & \frac{2}{3x} \\
\end{array}
\]
\[
(3x + 2)(2x + 3)
\]
Rule # 5: **Grouping Method** (4 terms)

Find the GCF 3 times:
- Arrange the terms into 2 pairs that share something
- Take out the GCF from each pair - parentheses should match
- Write down the GCF (parentheses) & the leftovers as factored pairs
- This method can also be used to solve trinomials

\[ r(a+b) + s(a+b) \]

Write down the GCF \( (a + b) \)
Write down the leftover \( (r + s) \)
Put together the 2 pieces \( (a + b)(r + s) \)

Factor: \( 21 - 7y + 3x - xy \)
Find pairs that share something: \( (21 - 7y)^\uparrow( 3x - xy) \)
Take out GCF of each pair: \( 7(3 - y) \) and \( x(3 - y) \)
Parentheses match if done correctly: \( (3 - y) \) & \( (3 - y) \)
Take GCF (parentheses) & write down left-over: \( (3 - y)(7 + x) \)

Factor: \( 4ax + 14ay - 10bx - 35by \)
\( (4ax + 14ay) + (-10bx - 35by) \)
If the 1st number in a ( ) is negative, 2a(2x+7y) - 5b(2x + 7y) TAKE out the (-)
\( (2x+7y)(2a-5b) \)
• This method can also be used to solve trinomials

Make a Product / Sum chart
Write down the 1st term
Write the 2 new parts in the middle
Write the last term
Now use the Grouping method to solve.

Factor: $x^2 - 8x + 15$

<table>
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<tbody>
<tr>
<td>$15x^2$</td>
<td>$-8x$</td>
</tr>
<tr>
<td>$-3x$</td>
<td></td>
</tr>
<tr>
<td>$-5x$</td>
<td></td>
</tr>
</tbody>
</table>

$x^2 - 3x - 5x + 15$

$(x^2 - 3x)(-5x + 15)$

$x(x - 3) - 5(x - 3)$

$(x - 5)(x - 3)$

Factor: $2x^2 + 13x - 7$

<table>
<thead>
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<th>Prod</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-14x^2$</td>
<td>$13x$</td>
</tr>
<tr>
<td>$14x$</td>
<td></td>
</tr>
</tbody>
</table>

$2x^2 + 14x - x - 7$

$(2x^2 + 14x)(x - 7)$

$2x(x + 7) - 1(x + 7)$

$(2x - 1)(x + 7)$
Rule # 6: **Factor Completely** (any # Terms)

- Factor until you can factor no more
- Use all 5 methods of factoring
- Repeat list (1-5) until you can’t factor anymore

**Factor: 6y^2 +27y -15**

*GCF:* 3

*Trinomial:* \((2y^2+9y-5) = (2y - 1)(y + 5)\)

*Answer:* 3(2y - 1)(y + 5)

**Factor: m^4 - 16**

*Diff 2 PSQ:* \((m^2 - 4)(m^2 + 4)\)

*D2PSQ:* \((m^2 - 4) = (m - 2)(m + 2)\)

*Answer:* \((m^2 + 4)(m - 2)(m + 2)\)

**Factor: 4m^4 + 8m^2 - 12**

*GCF:* 4

*Trinomial:* \((m^4 + 2m^2 - 3) = (m^2 - 1)(m^2 + 3)\)

*D2PSQ:* \((m^2 - 1) = (m - 1)(m + 1)\)

*Answer:* 4(m^2 + 3)(m - 1)(m + 1)
Strive towards Mental FOILing:

\[ y^2 + 8y + 15 \quad \text{Should be thinking what? times what? = 15 and adds/subtracts to be 8} \]

\[ (y + 3)(y + 5) \quad \text{FOIL quickly to check} \]

Practice:

\[ b^2 - 64 \quad (b - 8)(b + 8) \]

\[ y^2 - 3y - 10 \quad (y - 5)(y + 2) \]

\[ h^2 + 3h - 54 \quad (h + 9)(h - 6) \]

\[ 8f^3 - 27g^3 \quad (6x + 1)(x - 2) \]

\[ 6x^2 - 11x - 2 \quad (2f - 3g)(4f^2 + 6fg + 9g^2) \]