# AP Calculus AB/BC
## Summer 2023 Prep for Fall 2023

Name_____

Please complete all work in this packet.  Your teacher will give you a due date for the packet on the first day of class, but it will be advantageous for you to have it completed prior to the start of school as you will have homework each night in Calculus, starting with the first day of class. **SHOW ALL WORK**!  An answer alone without work may result in no credit given.  A **CALCULATOR is NOT** to be used while working on this packet.

The problems in this packet are designed to help you review topics that are important to your success in Calculus.  The problems must be done correctly, not just "tried."  Use your notes from previous math courses to help you, or you may visit the following websites for help:

https://www.khanacademy.org/
http://www.coolmath.com/precalculus-review-calculus-intro
http://mathforum.org/library/drmath/drmath.high.html

We will review these topics during the first two weeks of school.  There will then be a quiz covering these topics.  (An assignment sheet will be given on the first day of school with these dates on it.)  For those that miss any of the review days, you will still be required to take the quiz because the topics are review from previous classes.  We also will be giving a timed basic trig values quiz on the second day of class, so be prepared for that – know your trig values from special rights!

Calculus is relatively easy; it's the algebra that is difficult.  Most of the time, you will understand the calculus concept being taught, but will struggle to get the correct answers because of your background skills.  Please be diligent and figure out what you need help with so that we can clarify for you.  A little extra work this summer will go a long way to help you succeed in AP Calculus!

Print out a copy of the packet from The Woodlands High School website under the **Counselor Portal**; choose the **2023 AP Calculus AB/BC Summer Assignment**.  Print out your copy before summer starts and work on it continuously throughout the summer (a page a week).  DO NOT WAIT UNTIL THE DAY BEFORE SCHOOL STARTS TO WORK ON THIS!!!

We are looking forward to seeing all of you in the fall for an amazing year of Calculus with lots **of HIGH FIVES!!!  We think you will find this course challenging but enjoyable at the same time!**

Enjoy your summer and we will see you in the fall,


**Mrs. Gregory, Mrs. Frazee, & Mrs. Tatum**

## Important Trig Identities

| Pythagorean Relationships | Reciprocal Relationships | Double Angle Identities |
|---|---|---|
| $\sin^2 \theta + \cos^2 \theta = 1$ | $\tan x = \dfrac{\sin x}{\cos x} \qquad \cot x = \dfrac{\cos x}{\sin x}$ | $\sin 2x = 2\sin x \cos x$ |
| $1 + \tan^2 \theta = \sec^2 \theta$ | | |
| $1 + \cot^2 \theta = \csc^2 \theta$ | | $\cos 2x = \cos^2 x - \sin^2 x$ |
| | | $\cos 2x = 1 - 2\sin^2 x$ |
| | | $\cos 2x = 2\cos^2 x - 1$ |

➤ **Know all value**s **of** $\sin x$ and $\cos x$ (**quadrantals**) $[0,\ 2\pi]$

➤ **Know all Geometric Area Formulas** associated with: trapezoid, circle, semi-circle, cone, sphere, right circular cylinder, Pythagorean Theorem.

➤ **Exponents and Radicals:**          You must know the following rules.

$$a^0 = 1,\ \ a \neq 0 \qquad\qquad (ab)^x = a^x b^x \qquad\qquad a^x a^y = a^{x+y} \qquad\qquad \sqrt{a} = a^{1/2}$$

$$\frac{a^x}{a^y} = a^{x-y} \qquad\qquad \sqrt[n]{a} = a^{1/n} \qquad\qquad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \qquad\qquad \sqrt[n]{a^m} = a^{m/n}$$

$$a^{-x} = \frac{1}{a^x} \qquad\qquad \sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b} \qquad\qquad \left(a^x\right)^y = a^{xy} \qquad\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

**NO CALCULATORS** should be used.  Give exact values only.  IF a calculator should be used for a problem, it will be specified in that particular problem.

1.    a)  Given $g(x) = -x^2 + 3$, write the equation of the line through $g(-2)$ and $g(3)$.

      b)  Write the equation of the line that passes through the $x$-intercept of $2(3x - y) = -3$ and is perpendicular to this line.

Simplify.

2. $\dfrac{x^3-8}{x-2}$

3. $\dfrac{5-x}{x^2-25}$

4. $\dfrac{1}{x+h}-\dfrac{1}{x}$

5. $\dfrac{\dfrac{1}{3+x}-\dfrac{1}{3}}{x}$

6. $\left(4a^{5/3}\right)^{3/2}$

7. $\dfrac{4xy^{-2}}{12x^{-1/3}y^{-5}}$

8. $\left(\dfrac{x^7}{y^6}\right)^2\left(\dfrac{y^2}{x}\right)^4$

9. $a^2\left(a^{-1}+a^{-3}\right)$

10. $\left(\dfrac{a^6}{b^{-4}}\right)^0$

11. $\dfrac{3(n+1)!}{5n!}$

12. $\log_{1/2}8$

13. $e^{3\ln x}$

14. $3^{2\log_3 5}$

15. $\log_2 5+\log_2\left(x^2-1\right)-\log_2(x-1)$


Find all real solutions for $x$. Where applicable, give answer in interval notation.
**Give exact answers, no decimals.**

16. $x^2+3x-4=14$

17. $(x-5)^2=9$

18. $2x^2+5x=8$

19. $8x^3+27=0$

20. $x^6-16x^4=0$

21. $4x^3-8x^2-25x+50=0$

22. $27^{2x}=9^{x-3}$

23. $(x+1)^2(x-2)+(x+1)(x-2)^2=0$

24. $|2x+1|>3$

25. $|-x+4|\le 1$

26. $(x+3)(x-3)>0$

27. $x^2-2x-15\le 0$

28. $\dfrac{2x-1}{3x-2}\le 1$

29. $5^{(x+1)}=25$

30. $\dfrac{1}{3}=3^{2x+2}$

31. $e^{2x}=5$

32. $e^{-x}(8-x)=0$

33. $e^x(2-x)=0$

34. $\ln(x+3)=4$

35. $\log_3 x^2=2\log_3 4-4\log_3 5$

36. $2+\cos^2 x=3\sin^2 x;\quad 0\le x<2\pi$

37. $\tan x+\sec x=2\cos x;\quad -\infty<x<\infty$

Find the domain and range for the following functions.

38. $f(x) = \begin{cases} \sqrt{9-x^2}, & \text{if } 0 \le x < 3 \\ e^x, & \text{if } x < 0 \end{cases}$

39. $f(x) = \frac{\sin x}{x}$ (Use a calculator)

40. $f(x) = \frac{x^2 + x - 12}{x - 3}$
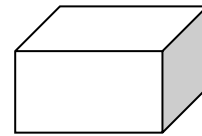
41. $f(x) = \sqrt{x^2 - 4}$

42. $f(x) = \ln(x-3)$

Expand using Log Properties.

43. $\ln\left(x^2 \sqrt{y}\right)$

44. $\log_3\left(\frac{x+3}{x^2}\right)$

Solve the following problems:

45. A closed box with a square base of side ($x$) and
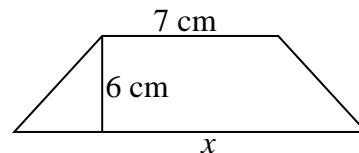height ($y$) has a surface area of 100 sq. ft.

   a) Express the volume of the box as a function of $x$.
      $V(x) =$

   b) Domain of $V(x)$      c) Range of $V(x)$

46. The area of the following trapezoid is $30\sqrt{2}$ cm$^2$.
Find the exact value of $x$.

7 cm

6 cm

$x$

47. Water is poured into a conical container with a diameter of 20 cm and a height of 38 cm. If the
volume of a cone is one-third of a cylinder, write an equation for the volume of water as a function of
the height of the water in the container.

20 cm

38 cm
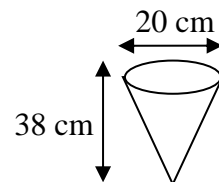
48. Given $f(x)=3x-5$ and $g(x)=2(x-4)^2+2x$, determine the following.

    a)      $g(f(x))$             b)      $f(g(3))$             c)      $f^{-1}(x)$

Find the inverse of the following functions:

49.      $f(x)=2x+3$             50.      $f(x)=\dfrac{x+2}{5x-1}$

Functions $f$ and $g$ are inverses of each other provided: $f(g(x))=x$ and $g(f(x))=x$.

51.      Verify that $f(x)=3x-5$ and $g(x)=\frac{1}{3}x+\frac{5}{3}$ are inverses of each other.

52.      Verify that $f(x)=2x+3$ and $g(x)=\frac{1}{2}x-\frac{3}{2}$ are inverses of each other.

You should be able to quickly evaluate the following trig expressions.

53.      $\sin\left(\dfrac{2\pi}{3}\right)$             54.      $\cos\left(-\dfrac{5\pi}{4}\right)$             55.      $\tan\left(\dfrac{11\pi}{6}\right)$

56.      $\csc\left(\dfrac{7\pi}{6}\right)$             57.      $\sec\left(\dfrac{5\pi}{3}\right)$             58.      $\tan\left(-\dfrac{\pi}{3}\right)$

59.      $\cos\left(\dfrac{3\pi}{4}\right)$             60.      $\cot\left(-\dfrac{\pi}{6}\right)$             61.      $\csc\left(\dfrac{4\pi}{3}\right)$

Simplify using trig identities.

62.      $\dfrac{1-\cos^2 x}{\sec^2 x-1}$             63.      $\sec x\cdot\sin 2x$             64.      $\dfrac{1}{1-\cos^2 x}$

Express the given number as a power of 2.

65.      $(64)^2$                                         66.      $\left(\dfrac{1}{8}\right)^3$

67.      $\left(2^4\cdot 16^{-2}\right)^3$                        68.      $\left(\dfrac{1}{2}\right)^{-8}\left(\dfrac{1}{4}\right)^4\left(\dfrac{1}{16}\right)^{-3}$

Factor and simplify.  Express your answer using only positive exponents.

69.     $3\left(x^2+4\right)^{\frac{4}{3}}+x\cdot4\left(x^2+4\right)^{\frac{1}{3}}\cdot2x$

70.     $2x\left(x^2+1\right)^{\frac{1}{2}}+x^2\cdot\dfrac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\cdot2x$

71.     $\dfrac{\left(x+4\right)^{\frac{1}{2}}-2x\left(x+4\right)^{-\frac{1}{2}}}{x+4}$

72.     $\dfrac{2x\left(1-x^2\right)^{\frac{1}{3}}+\dfrac{2}{3}x^3\left(1-x^2\right)^{-\frac{2}{3}}}{\left(1-x^2\right)^{\frac{2}{3}}}$
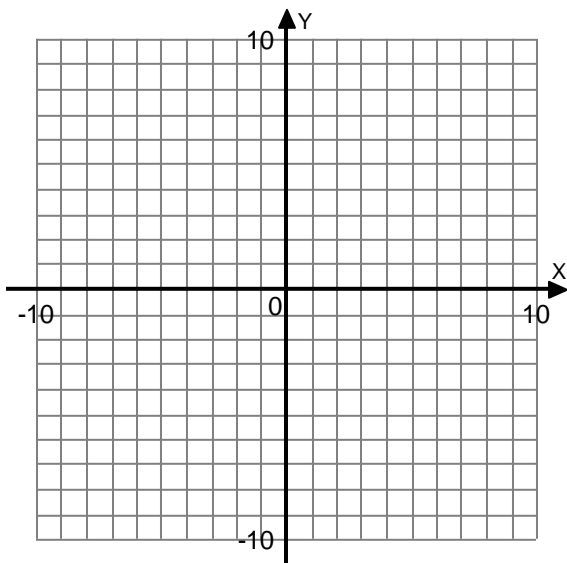
Graph the following functions, neatly on the graphs provided.

73.     $g(x)=\begin{cases} 2x+5, & \text{if } x>-3 \\ -2x-7, & \text{if } x\le-3 \end{cases}$

74.     $f(x)=\begin{cases} x^2, & \text{if } x<0 \\ x+2, & \text{if } 0\le x\le3 \\ 4, & \text{if } x>3 \end{cases}$
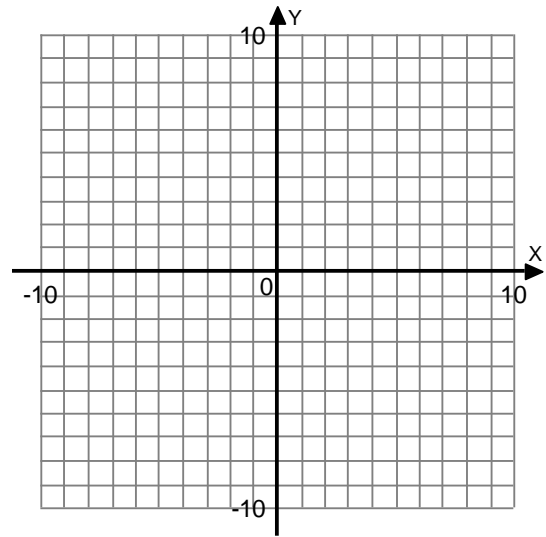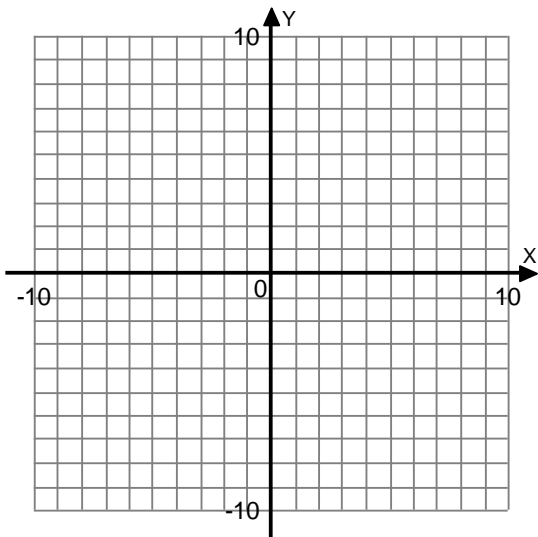




75.     Given $y=\dfrac{2(x+3)(x-2)}{(x-3)(x+3)}$,    graph the function and find the following:



a)   All holes _____

b)   All vertical asymptotes _____

c)   All horizontal asymptotes _____

d)   x-intercepts _____

e)   y-intercepts _____

The graphs of the following functions will be used routinely in the course. *Practice these graphs until you can demonstrate them from memory.* Graphs are given below and through the end for you to use.

76.    $y = x$                 77.    $y = \sqrt[3]{x}$          78.    $y = e^x$           79.    $y = x^2$

80.    $y = \cos x$         81.    $y = \dfrac{1}{x}$          82.    $y = \dfrac{1}{x^2}$        83.    $y = x^3$

84.    $y = \tan x$         85.    $y = |x|$           86.    $y = [\![x]\!]$         87.    $y = \sqrt{x}$

88.    $y = \ln x$         89.    $y = \sin x$         90.    $y = \pm\sqrt{1 - x^2}$     91.    $y = x^{2/3}$