TWHS AP Physics C:

Summer Assignment 2024

You are receiving this notice because you have elected to enroll in AP Physics C for the 2024 – 2025 school year.

Students who have selected AP Physics C as an alternate course are also receiving this notice.

The summer assignment is designed to help you review and prepare for AP Physics C. Read the assignment details below:

- You have been enrolled in a Canvas course titled "2024 AP Physics C Summer Assignment".
  - You must accept the course invitation to have access.
  - All of the information on this handout can also be found in the Canvas course.
- The purpose of the summer assignment is to review concepts from your junior-year physics course and to introduce a few new concepts that will be of importance in AP Physics C.
  - The first semester of AP Physics C covers mechanics (basically all of AP Physics 1 in one semester with more math).
  - So, reviewing previously-learned concepts is especially important since we will be moving very quickly and there are certain things you are expected to already know coming into the course (we will not be spending a lot of time reviewing).
  - **You are required to purchase the <u>AP Physics C Barron's Book</u> (newest edition, if possible) for this course.**
- The summer assignments will be graded; failure to complete them will not only have negative consequences for your grade, but will also cause you to start the year behind your peers (because you will be less prepared for the course).
- The majority of the summer 'units' follow the following format (some vary slightly):
  1. Reading & reading quiz
  2. Flipping Physics videos with embedded questions
  3. End-of-unit review quiz
- Please check the Canvas course regularly for any updates or additions.
- **<u>The very last section of the summer assignment is your notes for the first day of class – you MUST complete these prior to the first day of class!</u>**
  - The note guide is attached. If you lose this, it can also be found in the Canvas course.

The summer assignments will <u>unlock on Monday, June 3</u> and should be <u>completed BEFORE the first day of class</u>!!!!!

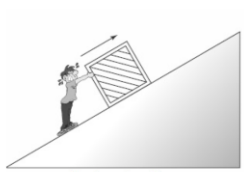
Please note: The assignments will be open and submittable for credit after the start of the school year (up to a certain date), but students who elect not to complete the assignments before the first day of class will be behind.

AP Physics C:

Introduction to Physics Calculus

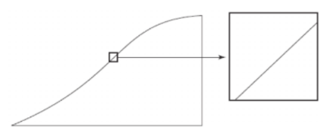
I. Derivatives of Polynomial Functions:

Calculus is just advanced algebra and geometry that has been tweaked to allow us to solve more sophisticated problems.

Algebra:	**Calculus:**
	
For the straight incline, the man pushes with an ***constant*** (or unchanging) force, and the crate goes up the incline at a ***constant*** (or unchanging) speed. With some simple physics formulas and regular math (including algebra and trig), you can compute how many calories of energy are required to push the crate up the incline.	On the other hand, for a curving incline, things are constantly ***changing***. The steepness of the incline is ***changing*** — and not just in increments like it's one steepness for the first 10 feet then a different steepness for the next 10 feet — it's constantly ***changing***. And the man pushes with a constantly ***changing*** force — the steeper the incline, the harder the push. As a result, the amount of energy expended is also ***changing***, not every second or every thousandth of a second, but constantly ***changing*** from one moment to the next. → Calculus allows us to deal with ***non-constant*** values!

Here's the cool part:

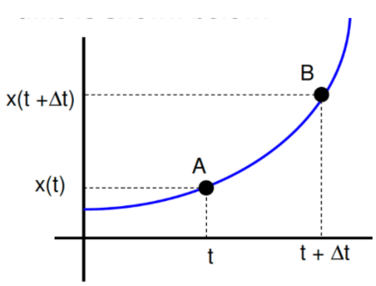
Calculus allows you to **ZOOM** in on a small part of the problem and apply the "regular" (basic algebra) math tools.



Calculus is About "Rates of Change"

- A ***rate*** is anything divided by ________________.
- ***Change*** is expressed using the Greek letter _________________________.
- Example: average velocity = the rate at which ___________________ changes ($\bar{v} = \frac{\Delta x}{\Delta t}$)

The graph below shows displacement as a function of time.

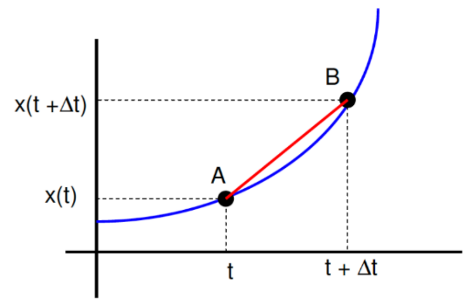


A time *t*, the object is at point A. While there, its position coordinate is x(t).

At time $t + \Delta t$, the object is located at point B. While there its position coordinate is x($t + \Delta t$).

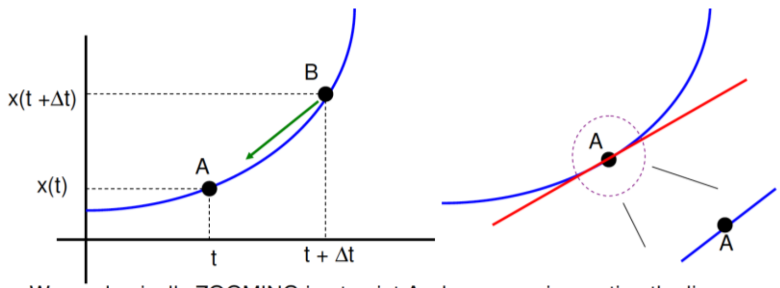
Suppose a secant line is drawn between points A and B.

(Note: the slope of the secant line is equal to the rise over the run)



READ THIS CAREFULLY:

If we hold point A fixed while allowing Δt to become very small, point B approaches point A and the secant approaches the tangent to the curve at point A.



We are basically ZOOMING in at point A where upon inspection the line "APPEARS" straight. Thus the secant line becomes a **TANGENT LINE**.

The Derivative:

Mathematically, we just found the slope!

"lim" stands for "limit" and it shows that Δt approaches zero, and as this happens the numerator approaches a finite number.

This is what a derivative is. A derivative yields a new function that defines the rate of change of the original function with respect to one of its variables. (In the above example, we see the rate of change of "x" with respect to time, "t".)

In most physics books the derivative is written like this:

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Mathematicians treat dx/dt as a single symbol which means "find the derivative". It is simply a mathematical operation.

THE DERIVATIVE IS THE SLOPE OF THE LINE TANGENT TO A POINT ON A CURVE.

Example 1:

Consider the function $x(t) = 3t + 2$.

What is the time rate of change of the function? I.e., what is the NEW FUNCTION that defines how $x(t)$ changes as t changes.

This is actually very easy – the entire equation is linear and looks like $y = mx + b$, thus we know from the beginning that the slope (the derivative) is equal to 3.

Nevertheless, we will follow through using the definition of the derivative:

* We didn't need to invoke the limit because the Δt's cancelled out.
* Regardless, we see that we get a constant.

Example 2: Consider the function $x(t) = t^2$.

The meaning:

For example, if t = 3 seconds, using $x(t) = t^2 = (3)^2 = 9$ meters.

The derivative, however, tells us how our displacement (x) changes as a function of time (t). The rate at which displacement changes is also called ***velocity***.

Thus, if we use our derivative we can find out how fast the object is traveling at t = 3 seconds. Since $dx/dt = 2t = 2(3) = 6$ m/s.

THERE IS A PATTERN HERE!!!!!!!

- If I had done the previous example with **t^3**, I would have gotten _____**.**
- If I had done the previous example with **t^4**, I would have gotten _____.
- If I had done the previous example with **t^5**, I would have gotten _____.
- And finally, if I had done the previous example with **t**, I would have gotten _____.

So, let's cheat: If $x(t) = kt^n$, then $\frac{dx}{dt}$ nkt^{n-1} → **THE POWER RULE**

Written on the Physics C equation sheet as: $$\frac{d(x^n)}{dx} = nx^{n-1}$$

Linear Graph → Ex.: y = 5x	Zero-Slope Line → Ex.: y = 5	Exponential Curve → Ex.: $5x^2$
The derivative is the slope of the line.	The derivative is zero because the slope is zero → the derivative of a constant is zero! (**THE CONSTANT RULE**)	The derivative is the slope of a line tangent to the curve.

Practice:

THE CONSTANT MULTIPLE RULE

1. If $x(t) = 20t^3$, then $\frac{dx}{dt} =$ ________________________.

THE SUM & DIFFERENCE RULE

2. If $x(t) = 5t^3 + 6t + 7$, then $\frac{dx}{dt} =$ ________________________.
3. If $x(t) = 2t^6 + 7t^4 + 4t + 2$, then $\frac{dx}{dt} =$ ________________________.

Other derivative rules we will encounter:

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

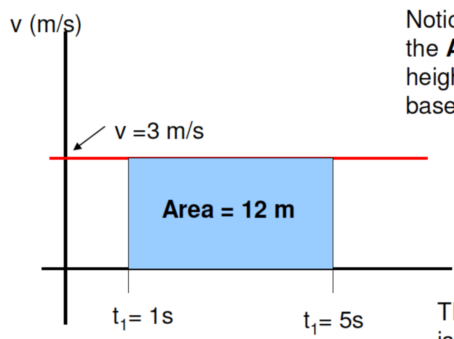
$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

II. *Integrals of Polynomial Functions:*

The "AREA"

You have learned that the rate of change of displacement is defined as the ***velocity*** of an object. Consider the graph below:

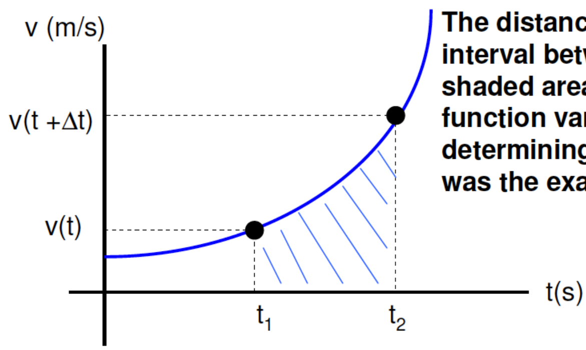
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \rightarrow v = \frac{dx}{dt}$$



Notice that the 12 m happens to be the **AREA** under the line or the height(v = 3 m/s) times the base(t= 4 seconds) = 12 meters

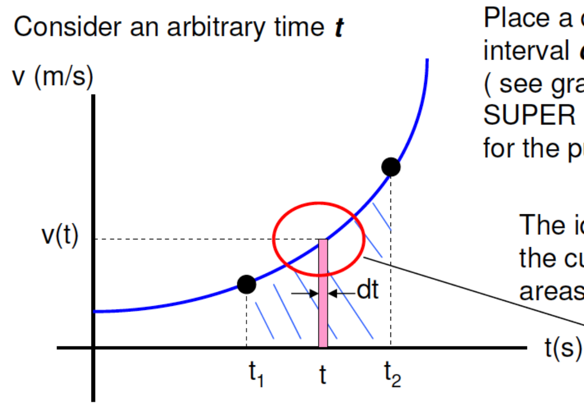
This works really nice if the function is linear. What if it isn't?

How do we determine how far something travels when the function is a curve? Consider the graph below:



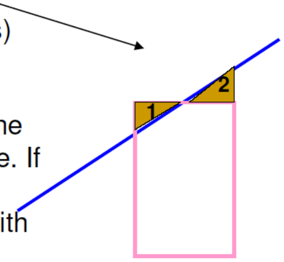
The distance traveled during the time interval between t_1 and t_2 equals the shaded area under the curve. As the function varies continuously, determining this area is NOT easy as was the example before.

So, once again, we **ZOOM** in:



Place a differential time interval ***dt*** about time ***t*** (see graph). This rectangle is SUPER SMALL and is only visible for the purpose of an explanation.

The idea is that the AREA under the curve is the SUM of all the areas of each individual "***dt***".



With "dt" very small, area 1 fits into area 2 so that the approximate area is simply the area of the rectangle. If we find this area for ALL the small ***dt's*** between ***t1*** and ***t2***, then added them all up, we would end up with the TOTAL AREA or TOTAL DISPLACEMENT.

The "INTEGRAL"

The temptation is to use the conventional summation sign "Σ". The problem is that you can only use Σ to denote the summing of discrete quantities and NOT for something that is continuously varying. Thus, we cannot use Σ.

When a continuous function is summed, a different sign is used. It is called an **integral** and looks like this: $\int$

When you are dealing with a situation when you have to integrate, realize:

1. We are given: the derivative.
2. We want to find: the original function x(t).

So basically, we are working backwards or finding the ANTI-DERIVATIVE.

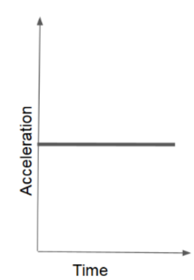
Definite Integral = has defined limits

Indefinite Integral = does not have defined limits

Example: An object is moving at a velocity with respect to time according to the equation v(t) = 2t.

a) What is the displacement function? (Hint: what was the ORIGINAL FUNCTION BEFORE the derivative was taken?)

- Taking the integral undoes the derivative, giving you the original function back. However, you can't get the constant back because the derivative of any constant is zero (constant rule of integration).
- "+C" represents all possible solutions (a general solution). For example, the only difference between f(x) = 2x + 1 and g(x) = 2x + 3 is the additive constant. (When integrating a function, you do not have enough info to determine which is correct, so we assume all are possible)
- Physics example: a(t) = 3 and since $a = dv/dt$ → $v(t) = \int a dt = 3t + C$ because we know $v(t) = at + v_0$.



b) How far did it travel from t = 2 seconds to t = 7 seconds?

You might have noticed that in the above example we had to find the **change(Δ)** over the integral to find the area, that is why we subtract. This might sound confusing. But integration does mean SUM. What we are doing is finding the TOTAL AREA from 0-7 and then the TOTAL AREA from 0-2. Then we can subtract the two numbers to get JUST THE AREA from 2-7.

Some other rules to keep in mind:

1. Constant rule → the integral of a constant : $\int adx = ax + C$
2. Zero rule → the derivative of any constant is zero, so the antiderivative (integral) of zero is all constants.
3. The constant multiple rule and the sum & difference rules we saw with derivatives also work in the same way with integrals

** A good way to check to see if you integrated correctly is to take the derivative of your answer – if you get the original function, your solution was correct! **

In Summary,

- **DERIVATIVES are used to find SLOPES!**
- **INTEGRALS are used to find AREAS!**

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$$

$$x = \int v\,dt \qquad v = \int a\,dt$$

Can we cheat?? YES!!!!

Power Rule for Integration:

$$\int x^n dx = \frac{1}{n+1} x^{n+1}$$

$(n \neq -1)$

Examples:

THE CONSTANT RULE

1. $y(x) = \int 6dx =$ ____________________

THE CONSTANT MULTIPLE

2. $x(t) = \int 3t^2 dt =$ ____________________

THE SUM & DIFFERENCE RULE

3. $y(x) = \int (x^3 + 2)dx =$ ____________________

Other integration rules we will encounter:

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$